Efficient Design of PN Supervisors with Disjunctive Specifications

Marian V. Iordache
Department of Engineering
LeTourneau University
Longview, TX 75607-7001

Po Wu, Feng Zhu, and Panos J. Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556

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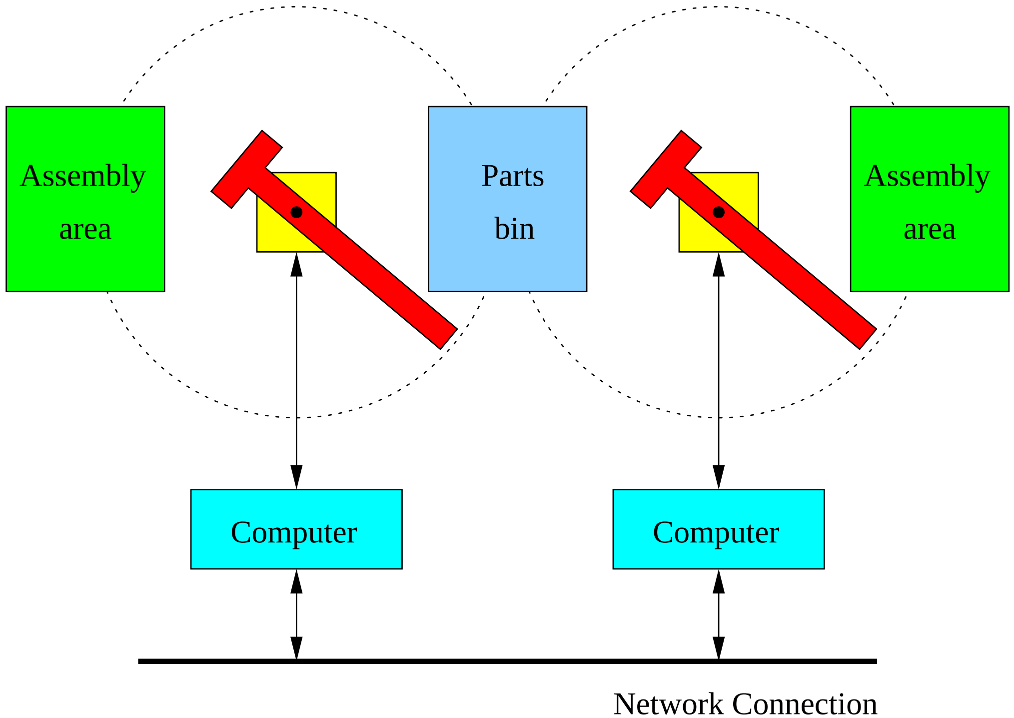
Constraints of the following form are easily enforced on a Petri net.

$$L\mu + Hq + Cv \leq b$$

- Any PN place constraints transition firings according to one such inequality.
- Any such inequality may be implemented by one place.

Utilization examples:

- $L\mu$ for mutual exclusion.
- Cv for fairness.
- Hq for enabling conditions.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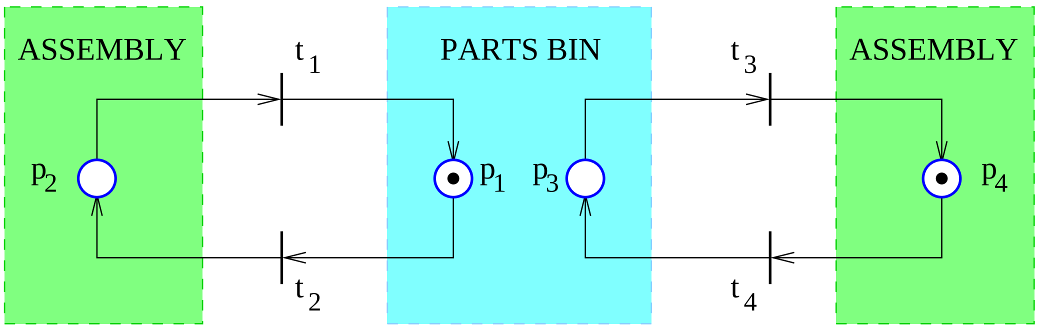
Network Connection


ASSEMBLY
PARTS BIN
ASSEMBLY
t 1
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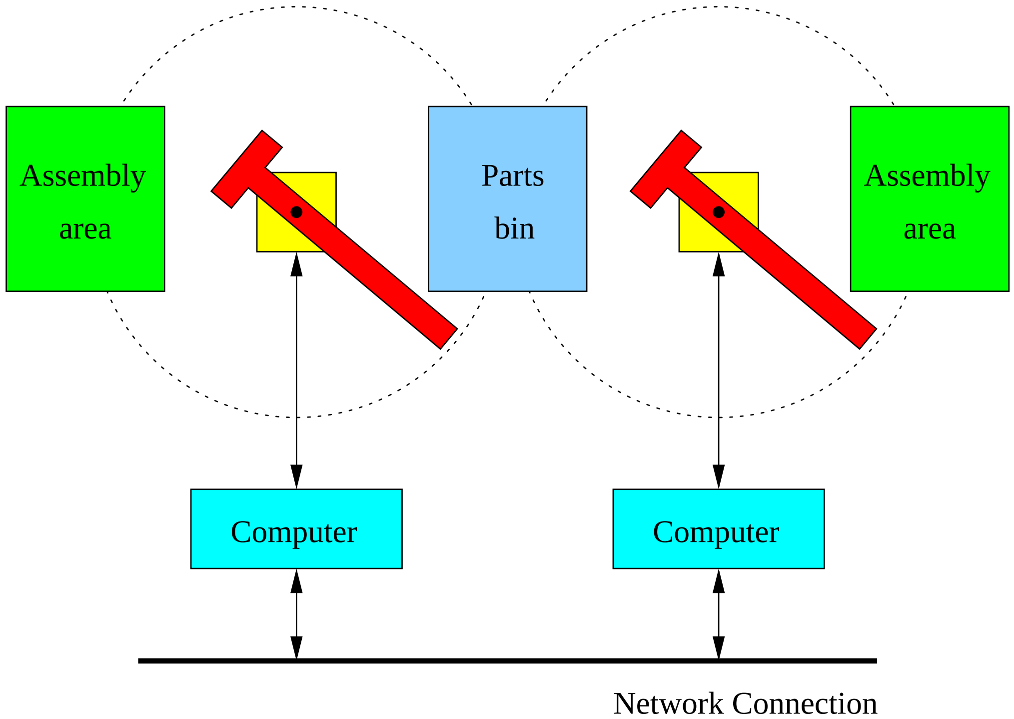
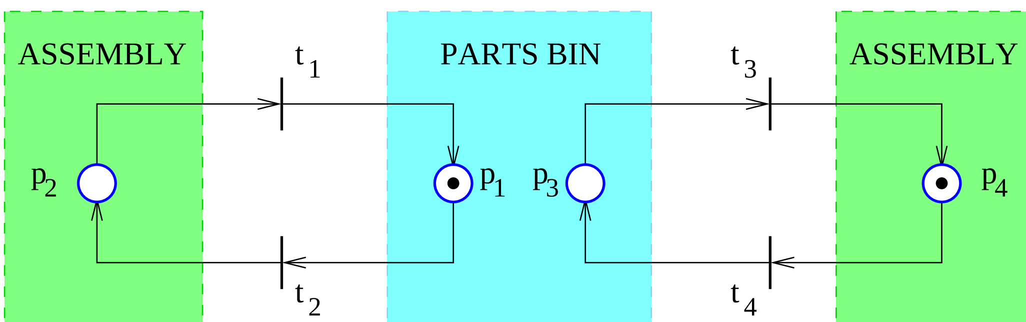
t 3
t 4
p2
p1
p3
p4

Motivation — Example





Mutual exclusion:

$$\mu_1 + \mu_3 \leq 1 \qquad (1)$$

Fairness:

$$v_1 - v_4 \leq n \qquad (2)$$

$$v_4 - v_1 \leq n \qquad (3)$$

Enabling condition:

$$q_1 + q_4 \leq \mu_{enable} \qquad (4)$$

A constraint that t_1 should be fired only if $\mu_3 = 0$ could be written as

$$q_1 \leq 0 \vee \mu_3 \leq 0$$

How to implement constraints involving disjunctions?

Note: With minor changes to the PN, the Cv term can be incorporated into the $L\mu$ term. From now on, let's focus on expressions

$$\boxed{L\mu + Hq \leq b}$$

Let l_i, h_i, and c_i denote the rows of L, H, and b.

$$L = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix} \qquad H = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{bmatrix} \qquad b = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

$L\mu + Hq \leq b$ may be written as

$$(l_1\mu + h_1 q \leq c_1) \wedge (l_2\mu + h_2 q \leq c_2) \wedge \ldots \wedge (l_n\mu + h_n q \leq c_n)$$

Disjunctive constraints of the form

$$(l_1\mu + h_1 q \leq c_1) \vee (l_2\mu + h_2 q \leq c_2) \vee \ldots \vee (l_n\mu + h_n q \leq c_n)$$

may be implemented as in [Iordache, 2007 ACC].

Difficulty: Conjunctions and disjunctions may be combined, as in

$$(L_1\mu + H_1 q \leq b_1) \vee (L_2\mu + H_2 q \leq b_2) \vee \ldots \vee (L_n\mu + H_n q \leq b_n)$$

The logic expression has to be converted to the conjunctive normal form in order to apply the method of [Iordache, 2007 ACC].

A conversion to the conjunctive normal form may result in an exponential increase in the number of terms!

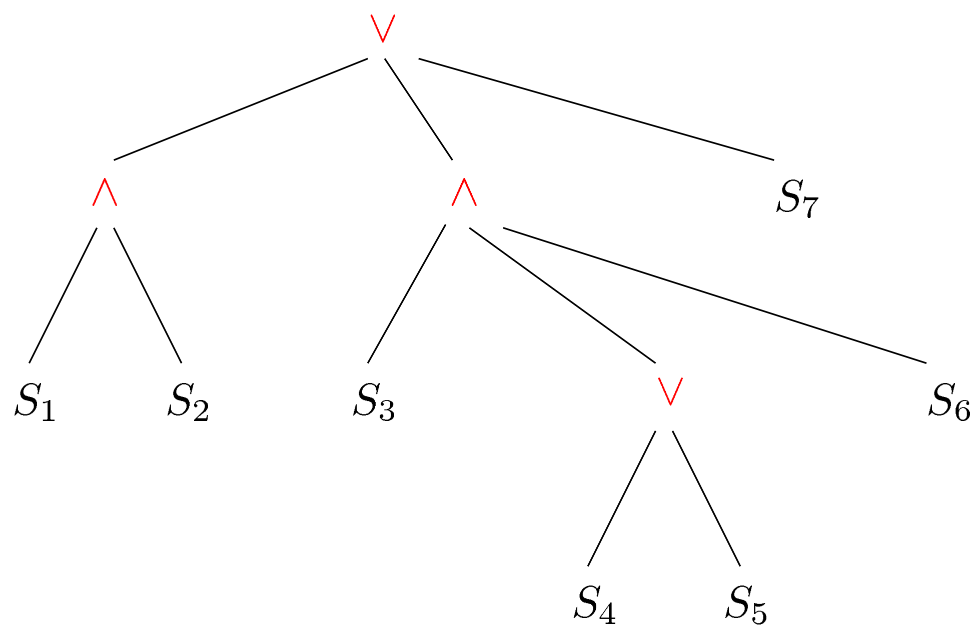
Contribution

This paper extends the method of [Iordache, 2007 ACC] so as to avoid conversions to the conjunctive normal form.

The specification may be expressed by arbitrary logic expressions involving disjunctions and conjunctions.

Let S_i denote the predicate $l_i\mu + h_i q \leq c_i$.

Specifications are represented by a tree indicating how the predicates S_i are combined.



$$E = (S_1 \wedge S_2) \vee (S_3 \wedge (S_4 \vee S_5) \wedge S_6) \vee S_7$$

Eliminate the firing vector terms by replacing $l_i\mu + h_i q \leq c_i$ in terms of the PN $\mathcal{N}$ with $l_i^*\mu^* \leq c_i$ in terms of the PN $\mathcal{N}^*$.

Let δ_i be a variable equal to the truth value of the predicate S_i^*.

Recursively combine the leaf nodes into single nodes:

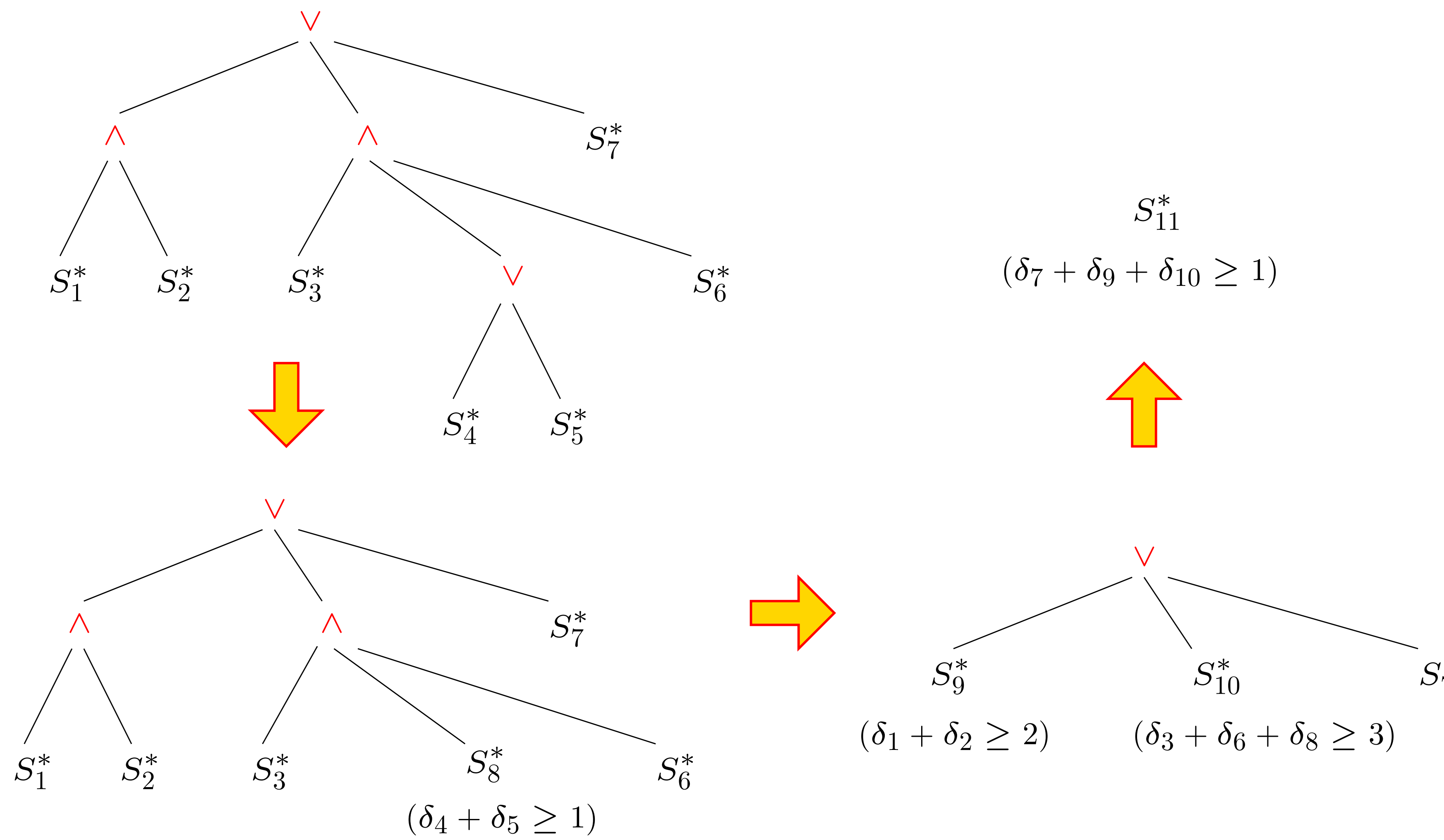
- Replace $S_1^* \vee S_2^* \vee \ldots \vee S_n^*$ with S_z^* standing for $\delta_1 + \delta_2 + \ldots + \delta_n \geq 1$.

- Replace $S_1^* \wedge S_2^* \wedge \ldots \wedge S_n^*$ with S_z^* standing for $\delta_1 + \delta_2 + \ldots + \delta_n \geq n$.

Under *certain boundedness assumptions*, the constraint that δ_i is the truth value of S_i^* can be expressed by marking inequalities.

Enforce all inequalities in order to obtain the closed-loop PN.

Solution

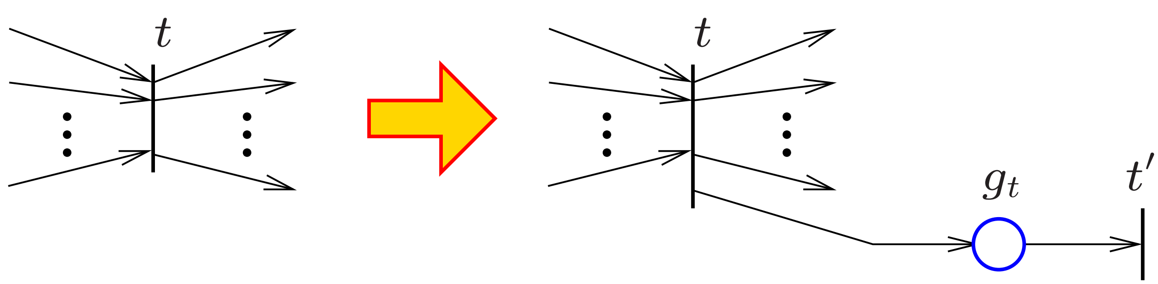

∨
∧
∧
S_7^*
S_1^*
S_2^*
S_3^*
∨
S_6^*
S_4^*
S_5^*
S_{11}^*
$(\delta_7 + \delta_9 + \delta_{10} \geq 1)$
∨
∧
∧
S_7^*
S_1^*
S_2^*
S_3^*
S_8^*
$(\delta_4 + \delta_5 \geq 1)$
S_6^*
∨
S_9^*
$(\delta_1 + \delta_2 \geq 2)$
S_{10}^*
$(\delta_3 + \delta_6 + \delta_8 \geq 3)$
S_7^*

Replace $\mathcal{N}$ with $\mathcal{N}^*$ and convert $l_i\mu + h_iq \leq c_i$ to $l_i^*\mu^* \leq c_i$ as follows.

Let $h_{d,i} = \max(0, l_iD, h_i)$, where D is the incidence matrix.

Let $T_s = \{t \in T : \exists i,\, l_iD(\cdot,t) \neq h_{d,i}(t)\}$.

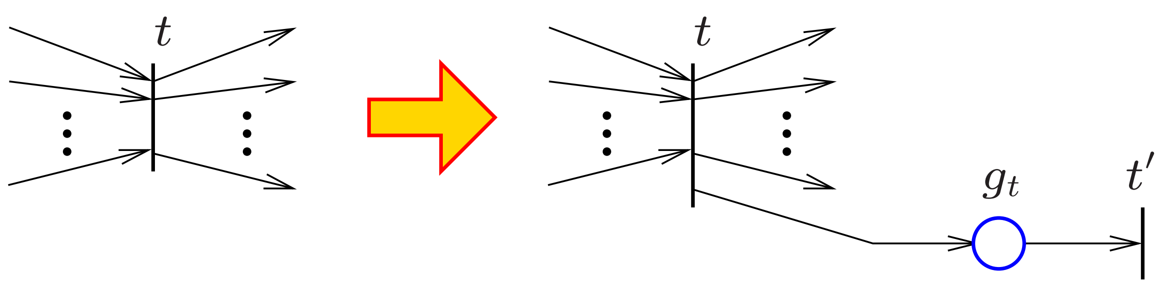
For all transitions $t \in T_s$:



$$\forall p \in P:\ l_i^*(p) \quad = \quad l_i(p) \tag{5}$$

$$\forall t \in T_s:\ l_i^*(g_t) \quad = \quad h_{d,i}(t) - l_iD(\cdot,t). \tag{6}$$

For all transitions $t \in T_s$:



$$\forall p \in P: \; l_i^*(p) = l_i(p)$$
$$\forall t \in T_s: \; l_i^*(g_t) = h_{d,i}(t) - l_i D(\cdot, t).$$

Proposition. *If $\forall p \in P : \mu^*(p) = \mu(p)$, $\forall t \in T_s : \mu^*(g_t) = 0$, and for some $t \in T_s$, $\mu \xrightarrow{t} \mu_1$ in $\mathcal{N}$ and $\mu^* \xrightarrow{t} \mu_0^* \xrightarrow{t'} \mu_1^*$ in $\mathcal{N}^*$, then $l_i^* \mu^* = l_i \mu$, $l_i^* \mu_0^* = l_i \mu + h_{d,i} q$, and $l_i^* \mu_1^* = l_i \mu_1$, where q is the firing vector when t is fired.*

Assume $l_i^* \mu^*$ has known bounds m_i and M_i: $m_i \leq l_i^* \mu^* \leq M_i$.

Let $\delta_i \in \{0, 1\}$.

δ_i is constrained to equal the truth value of $l_i^* \mu^* \leq c_i$ as follows.

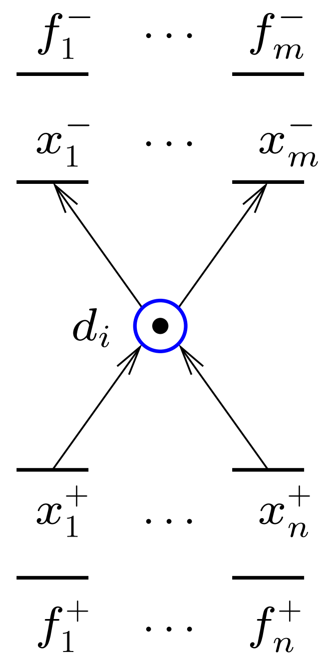
- To ensure that $l_i^* \mu^* > c_i \Rightarrow \delta_i = 0$ write

$$l_i^* \mu^* \leq c_i \delta_i + M_i (1 - \delta_i) \tag{7}$$

- To ensure that $l_i^* \mu^* \leq c_i \Rightarrow \delta_i = 1$ write

$$l_i^* \mu^* \geq m_i \delta_i + (c_i + 1)(1 - \delta_i) \tag{8}$$

The *predicate net* of $l_i^* \mu^* \leq c_i$ w.r.t. the PN $(\mathcal{N}^*, \mu_0^*)$ is defined as follows.



Let $\delta_i = \mu(d_i)$.

The initial value of δ_i: the truth value of $l_i^* \mu_0^* \leq c_i$.

δ_i will be constrained by:

$$l_i^* \mu^* \leq c_i \delta_i + M_i(1 - \delta_i) \quad (9)$$

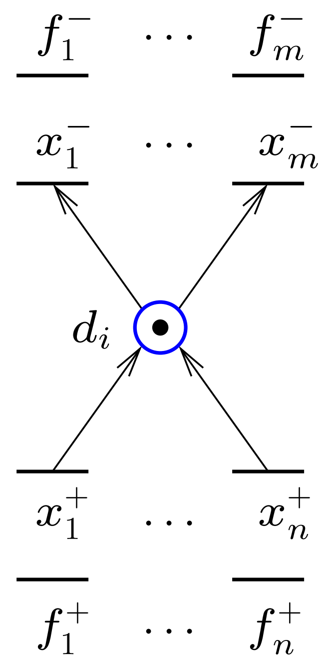
$$l_i^* \mu^* \geq m_i \delta_i + (c_i + 1)(1 - \delta_i) \quad (10)$$

To ensure $\delta_i \in \{0, 1\}$ (by mistake, not in the paper):

$$\delta_i \leq 1 \quad (11)$$

(The constraint above is needed when $M_i - m_i \geq 2(M_i - c_i)$.)

Let $t_1^+ \dots t_n^+$ ($t_1^- \dots t_m^-$) be the transitions that increase (decrease) $c_i - l_i^* \mu^*$.



Let ρ be a function associating a unique label to each transition of $\mathcal{N}^*$.

For each $t \in \{t_1^+, \dots, t_n^+, t_1^-, \dots, t_m^-\}$, create the transitions f and x of label $\rho(t)$.

The constraints (9)–(11) do not restrict t.

The constraints (9)–(11) ensure that when t fires:

- f is fired if δ_i should not change.
- x is fired if δ_i should change.

Let $\mathcal{L}$ be a set of constraints. Initialize $\mathcal{L} = \emptyset$.

Recursively combine the leaf nodes into single nodes:

- Replace $S_1^* \vee S_2^* \vee \ldots \vee S_n^*$ with S_z^* standing for $\delta_1 + \delta_2 + \ldots + \delta_n \geq 1$.
- Replace $S_1^* \wedge S_2^* \wedge \ldots \wedge S_n^*$ with S_z^* standing for $\delta_1 + \delta_2 + \ldots + \delta_n \geq n$.
- Let $\mathcal{N}_z^*$ be the predicate net of S_z^*.
- Let $\mathcal{N}^* = \mathcal{N}^* \| \mathcal{N}_1^* \| \ldots \| \mathcal{N}_n^*$.
- For every S_z^*, record the inequalities (9)–(11) in $\mathcal{L}$.

Enforce the inequality of the root node and the inequalities in $\mathcal{L}$.

Theorem. *Consider the closed-loop PN. If the specification is satisfied at the initial marking, it is satisfied also for all reachable markings. Moreover, assuming that all supervisor transitions are fired as soon as enabled and that the plant does not fire multiple transitions at the same time, the supervision is least restrictive.*

The two assumptions are needed for a least restrictive supervision.

The *no concurrency* assumption is needed in order to ensure that the constraints (9)–(11) of the predicate nets do not restrict the plant.

Conclusion

The paper shows how to enforce specifications consisting of arbitrary disjunctions and conjunctions of linear inequalities.

The closed-loop system is still a Petri net.

The supervision is least restrictive.

Certain boundedness assumptions are made.

The complexity of our previous method is considerably reduced.