Synthesis of Supervisors Enforcing General Linear Vector Constraints in Petri Nets

Marian V. Iordache and **Panos J. Antsaklis**
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556
iordache.1, antsaklis.1@nd.edu

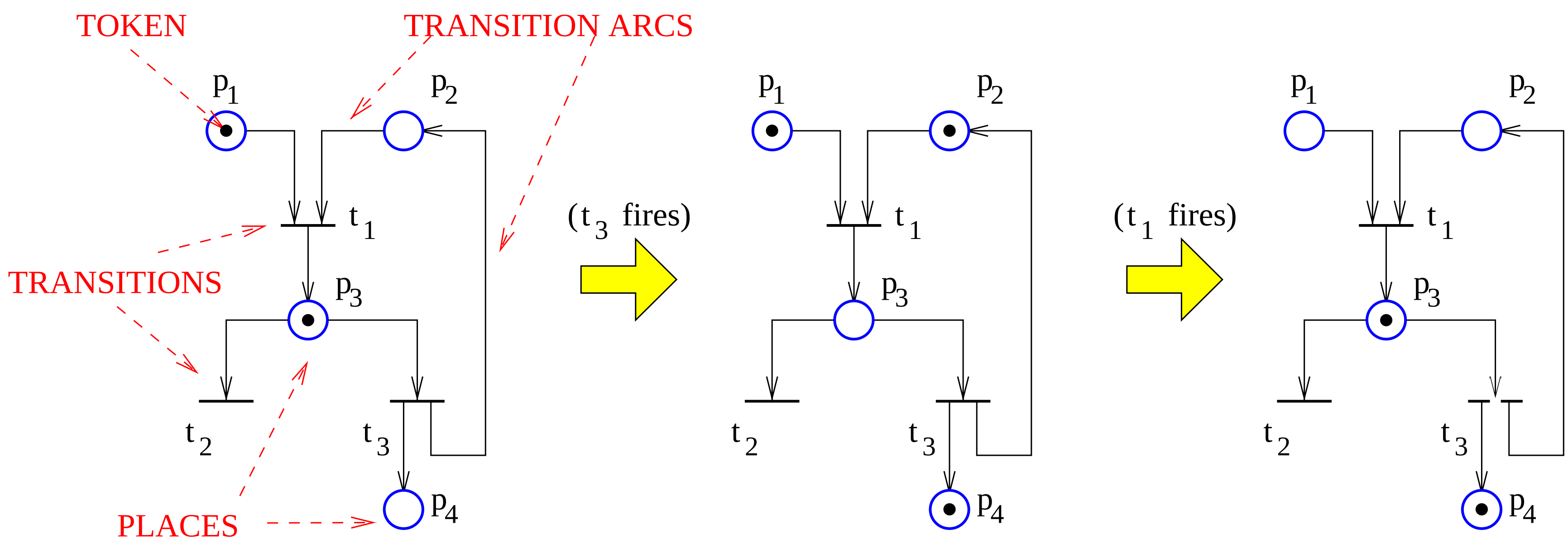
Outline

- Notation
- Description of the constraints
- Generality of the constraints
- Supervisor design for fully controllable and observable PNs
- Supervisor design for partially controllable and observable PNs
- Example
- Final Remarks

Notation

Notation: μ – the marking, μ_0 – the initial marking, D – the incidence matrix, q – the firing vector, and v – the Parikh vector. Let μ_i denote $\mu(p_i)$ and v_j denote $v(t_j)$.

The state equation: $\mu = \mu_0 + Dv$.



$$\mu_0 = [\ 1 \quad 0 \quad 1 \quad 0\ ]^T \qquad \mu' = [\ 1 \quad 1 \quad 0 \quad 1\ ]^T \qquad \mu'' = [\ 0 \quad 0 \quad 1 \quad 1\ ]^T$$

$$v = [\ 0 \quad 0 \quad 0\ ]^T \qquad v = [\ 0 \quad 0 \quad 1\ ]^T \qquad v = [\ 1 \quad 0 \quad 1\ ]^T$$

$$q = [\ 0 \quad 0 \quad 1\ ]^T \qquad q = [\ 1 \quad 0 \quad 0\ ]^T$$

Constraint Description

This paper shows that *generalized linear constraints*, involving the marking, the firing vector and the Parikh vector can be enforced as effectively as the linear marking constraints.

The following are defined:

1. Linear Marking Constraints
2. Constraints involving the firing vector and the marking
3. The generalized linear constraints

Constraint Description

1. *Linear Marking Constraints* (also known as *Generalized Mutual Exclusion Constraints*):

$$L\mu \leq b \tag{1}$$

This requires the initial marking μ_0 to satisfy

$$L\mu_0 \leq b$$

and that a transition t may fire from a marking μ iff

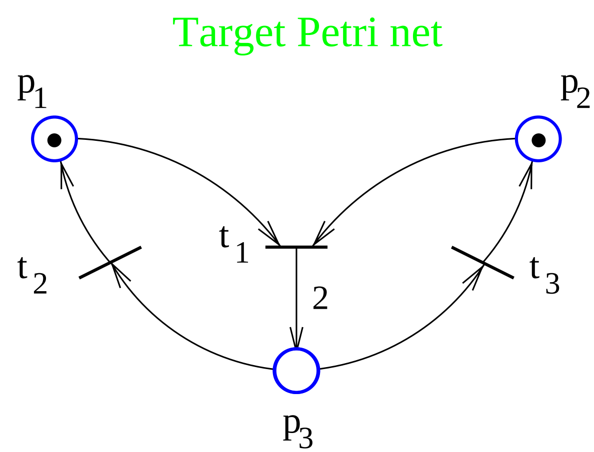
(a) $\mu \xrightarrow{t} \mu'$
(b) $L\mu' \leq b$

In the literature, linear marking constraints have been used to represent

1. Logical constraints.
2. Mutual exclusion.
3. Markings for which a PN is deadlock-free/live.

Let $L\mu \leq b$ be $\mu_1 + \mu_3 \geq 1$. Then:

$$L = \begin{bmatrix} -1 & 0 & -1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \end{bmatrix}$$

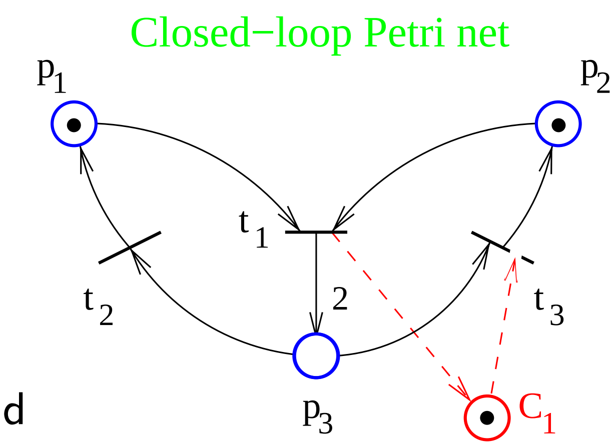


The incidence matrix is:

$$D = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{bmatrix}$$

The supervisor has one control place (as L has one row):

$$D_s = -LD = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$



The initial marking of the supervisor is

$$\mu_{s0} = b - L\mu_0 = \begin{bmatrix} 1 \end{bmatrix}$$

As $\mu_s = b - L\mu$ for all reachable markings μ, the method is called *supervision based on place invariants*.

Constraint Description

2. *Constraints involving the marking and the firing vector*:

$$L\mu + Hq \leq b \tag{2}$$

This requires the initial marking μ_0 to satisfy

$$L\mu_0 \leq b$$

and that a transition t_i may fire from a marking μ iff

(a) $\mu \xrightarrow{t_i} \mu'$

(b) $L\mu' \leq b$

(c) $L\mu + Hq \leq b$ for $q(i) = 1$ and $q(j) = 0 \ \forall j \neq i$.

In the literature, constraints involving the firing vector have been used for

1. Routing in communication networks.
2. The control of railway networks.

Constraint Description

3. *The generalized linear constraints*:

$$L\mu + Hq + Cv \leq b \tag{3}$$

This requires the initial marking μ_0 to satisfy

$$L\mu_0 \leq b$$

and that a transition t_i may fire from a current state (μ, v) iff

(a) $\mu \xrightarrow{t_i} \mu'$
(b) $L\mu + Hq + Cv \leq b$ for $q(i) = 1$ and $q(j) = 0 \; \forall j \neq i$.
(c) $L\mu' + Cv' \leq b$, where $v' = v + q$.

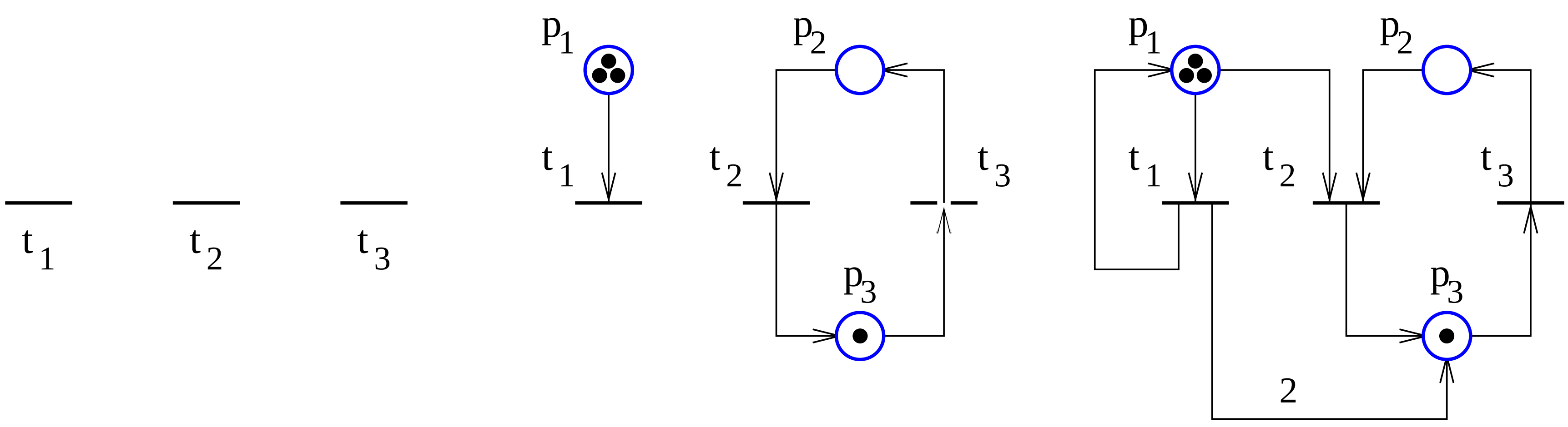
Constraint Generality

Application of generalized linear constraints:

1. In the literature, the simpler constraints $Cv \leq b$ have been used to specify fairness constraints.
2. We show that any supervisor designed as *control places* arbitrarily connected to the places of a PN can be described as enforcing constraints $Hq + Cv \leq b$.
3. We show on an AGV coordination example how constraints in the generalized linear form can naturally arise.

Constraint Generality

The places of any PN can be seen as control places enforcing (3):



(a) (b) (c)

unconstrained operation

$$
\begin{array}{lrcl}
(p_1) & v_1 & \leq & 3 \\
(p_2) & v_2 - v_3 & \leq & 0 \\
(p_3) & -v_2 + v_3 & \leq & 1
\end{array}
\qquad
\begin{array}{rcl}
q_1 + v_2 & \leq & 3 \\
v_2 - v_3 & \leq & 0 \\
-2v_1 - v_2 + v_3 & \leq & 1
\end{array}
$$

Given $L\mu + Hq + Cv \leq b$, let:

$$D_{lc}^{+} = \max(0, -LD - C) \quad (4)$$
$$D_{lc}^{-} = \max(0, LD + C) \quad (5)$$

The supervisor is given by the incidence matrices:

$$D_{c}^{+} = D_{lc}^{+} + \max(0, H - D_{lc}^{-}) \quad (6)$$
$$D_{c}^{-} = \max(D_{lc}^{-}, H) \quad (7)$$

The initial marking of the supervisor is:

$$\mu_{c0} = b - L\mu_0 \quad (8)$$

Theorem 1. *The supervisor defined by the input and output matrices* D_c^+ *and* D_c^- *and of initial marking* μ_{c0}*, enforces* $L\mu + Hq + Cv \leq b$ *and is least restrictive.*

A set of constraints $L\mu + Hq + Cv \leq b$ is said to be ***admissible*** if the approach for fully controllable and observable PNs generates a supervisor which never attempts to inhibit plant-enabled uncontrollable transitions and detect closed-loop-enabled unobservable transitions.

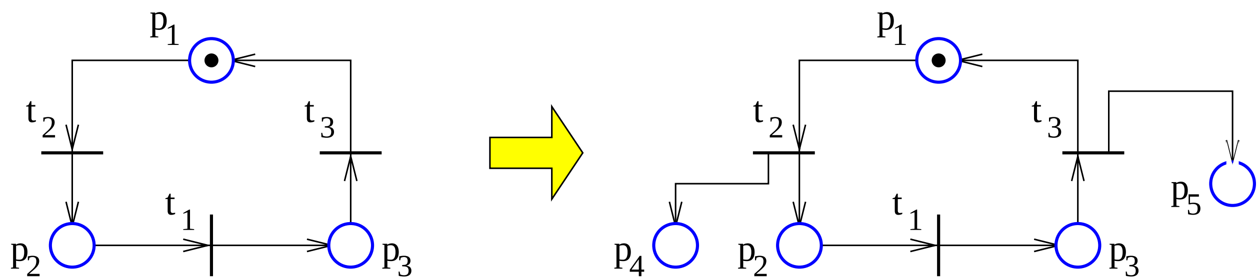
If $L\mu + Hq + Cv \leq b$ is not admissible, our approach is to find a set of constraints $L_a\mu + H_aq + C_av \leq b$ such that

1. $L_a\mu + H_aq + C_av \leq b \Rightarrow L\mu + Hq + Cv \leq b$
2. $L_a\mu + H_aq + C_av \leq b$ *is admissible.*

Effective techniques exist for enforcing constraints $L\mu \leq b$ in partially controllable and observable PNs.

Our approach transforms the problem of enforcing $L\mu + Hq + Cv \leq b$ into the problem of enforcing $L_t\mu_t \leq b$ in a transformed PN.

Illustration of the *C-Transformation*:



This transformation maps

$$\mu_1 + q_1 + v_2 - v_3 \leq 3 \tag{9}$$

into

$$\mu_1 + q_1 + \mu_4 - \mu_5 \leq 3 \tag{10}$$

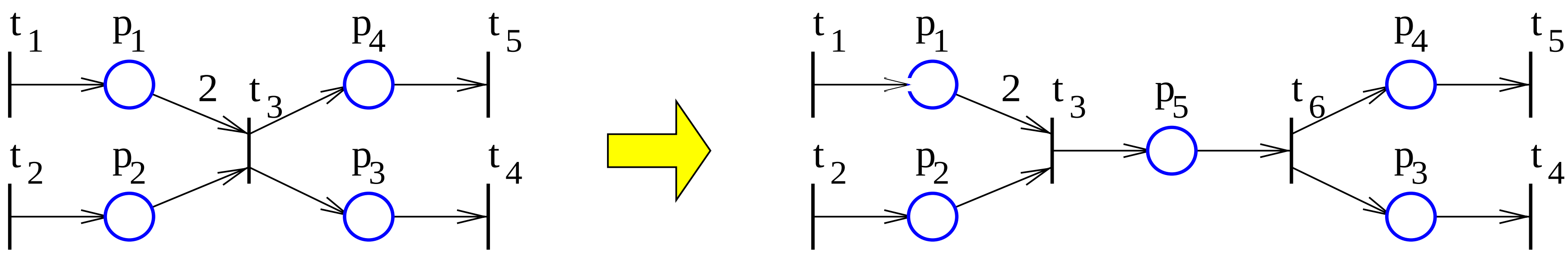
The inverse transformation is possible and maps

$$\mu_1 - 3\mu_4 + 2\mu_5 + q_1 \leq 5 \tag{11}$$

into

$$\mu_1 + q_1 - 3v_2 + 2v_3 \leq 5 \tag{12}$$

Illustration of the *H-Transformation*:



This transformation maps

$$\mu_1 + \mu_2 + 2\mu_3 + q_3 \leq 5 \qquad (13)$$

into

$$\mu_1 + \mu_2 + 2\mu_3 + 4\mu_5 \leq 5 \qquad (14)$$

The term $4\mu_5$ is obtained as follows. Consider firing t_3 in the transformed net: $\mu \xrightarrow{t_3} \mu'$. The coefficient a of t_3 is to satisfy that

$$a + \mu_1' + \mu_2' + 2\mu_3' = 1 + \mu_1 + \mu_2 + 2\mu_3$$

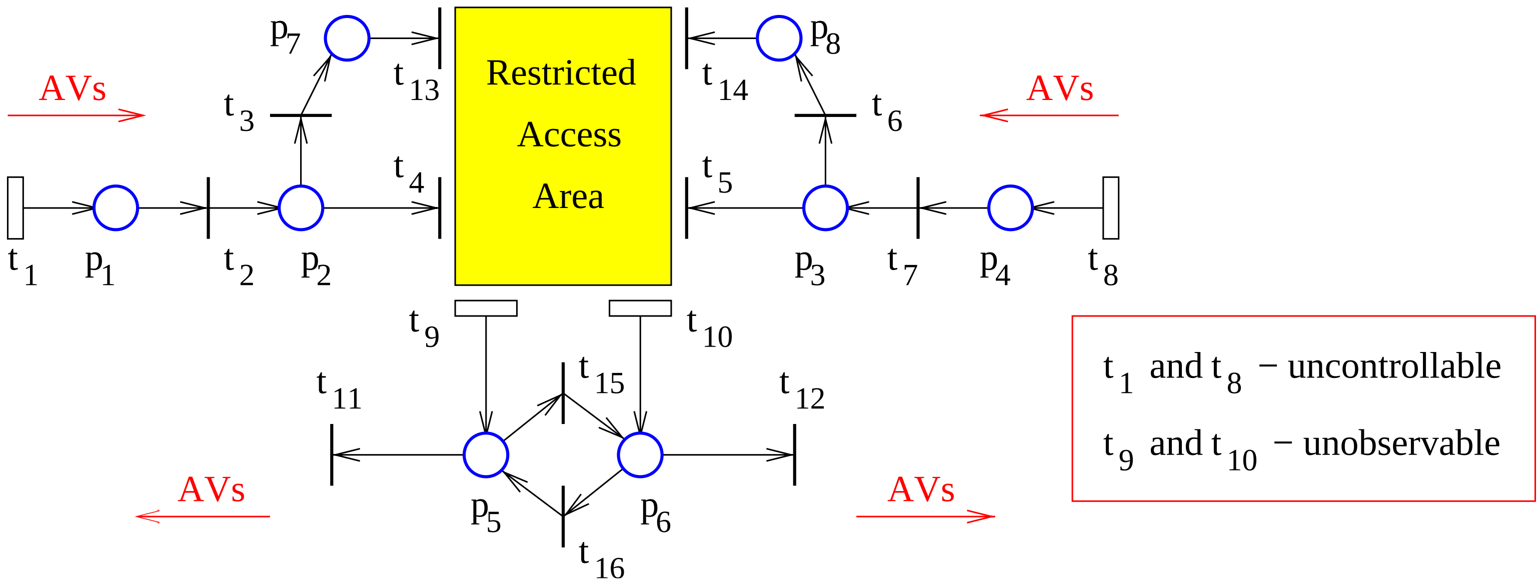
The inverse transformation can also be defined.

Given the PN $\mathcal{N}$ and the set of constraints $L\mu + Hq + Cv \leq b$:

1. Apply the *C-transformation* and then the *H-transformation*. This maps $\mathcal{N}$ to $\mathcal{N}_{HC}$ and $L\mu + Hq + Cv \leq b$ to $L_{HC}\mu \leq b$.
2. Test whether $L_{HC}\mu_{HC} \leq b$ is admissible in $\mathcal{N}_{HC}$. If so, exit, and declare $L\mu + Hq + Cv \leq b$ admissible.
3. *Find a set of admissible constraints $L_{HCa}\mu_{HC} \leq b_a$ such that $L_{HCa}\mu_{HC} \leq b_a \Rightarrow L_{HC}\mu_{HC} \leq b$. In case of failure, exit and declare failure to find admissible constraints.*
4. Apply the *inverse H-* and *C-transformations*. This maps $L_{HCa}\mu_{HC} \leq b_a$ to $L_a\mu + H_aq + C_av \leq b_a$.

Theorem 2. *$L_a\mu + H_aq + C_av \leq b_a$ is admissible, and a supervisor enforcing it enforces also $L\mu + Hq + Cv \leq b$ (that is, $L_a\mu + H_aq + C_av \leq b_a \Rightarrow L\mu + Hq + Cv \leq b$.)*

Example



The number of AVs in the RA is $v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}$ and is limited to m.

It is necessary to wait for arbitration when the number of AVs in the RA is $m - 1$ and both a left and a right vehicle attempt to enter the RA.

AVs should not wait for arbitration otherwise.

The arbitration is to be fair (not to favor left or right AVs).

$$2q_5 + \mu_2 + \mu_7 \leq m - (v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) + 1 \text{ (inadmissible)} \quad (15)$$

$$2q_4 + \mu_3 + \mu_8 \leq m - (v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) + 1 \text{ (inadmissible)} \quad (16)$$

$$mq_3 \leq \mu_3 + \mu_8 + v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} \text{ (impossible)} \quad (17)$$

$$mq_6 \leq \mu_2 + \mu_7 + v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} \text{ (impossible)} \quad (18)$$

$$\mu_2 + \mu_7 \leq 1 \text{ (admissible)} \quad (19)$$

$$\mu_3 + \mu_8 \leq 1 \text{ (admissible)} \quad (20)$$

The requirement on the maximum number of AVs in the RA is

$$v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} \leq m \text{ (inadmissible)} \quad (21)$$

Fairness constraints:

$$v_3 - v_6 \leq n \text{ (admissible)} \quad (22)$$

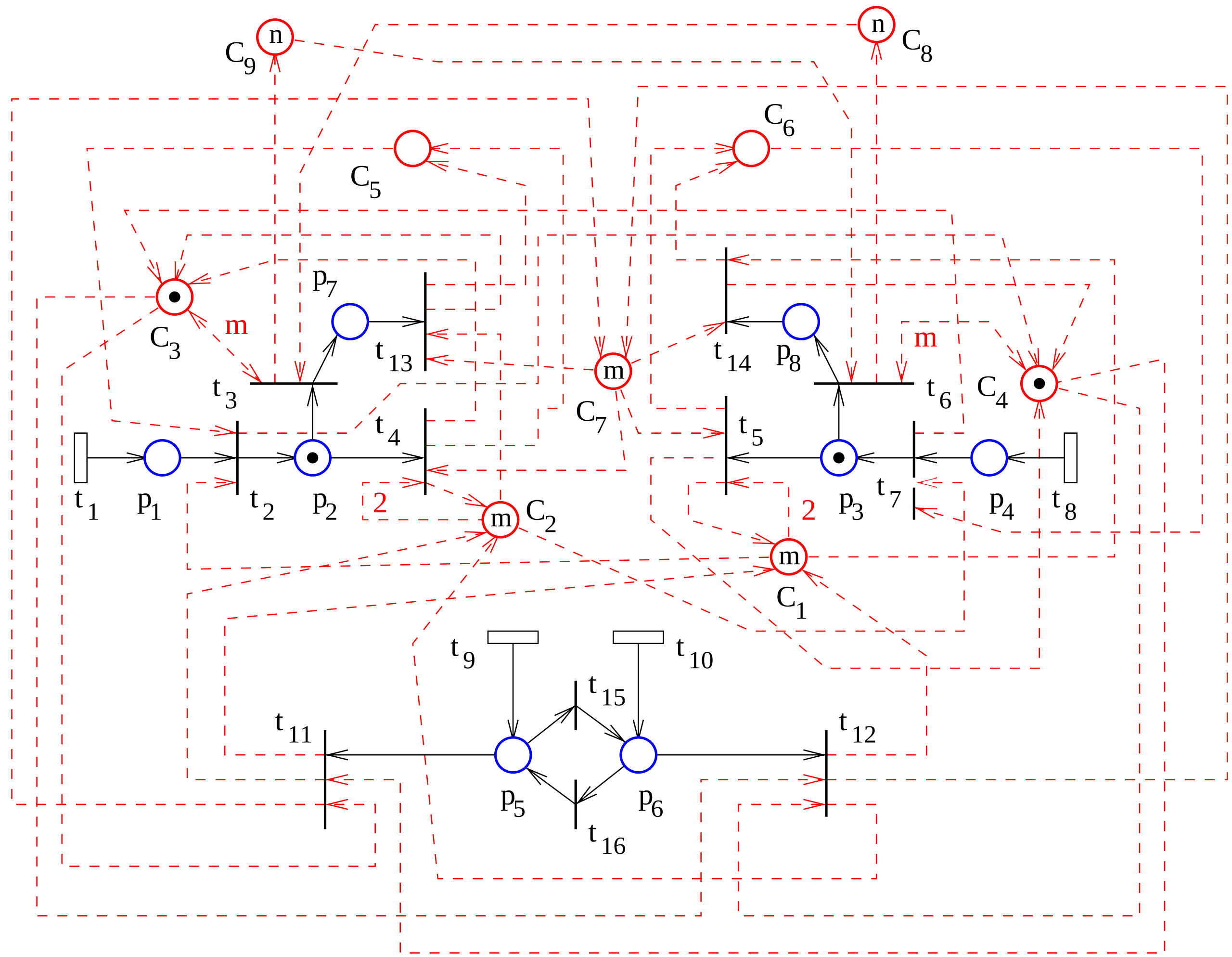
$$-v_3 + v_6 \leq n \text{ (admissible)} \quad (23)$$

Transformed constraints

$$2q_5 + \mu_2 + \mu_5 + \mu_6 + \mu_7 + v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} \leq m + 1 \quad (24)$$
$$2q_4 + \mu_3 + \mu_5 + \mu_6 + \mu_8 + v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} \leq m + 1 \quad (25)$$
$$v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10} + \mu_5 + \mu_6 \leq m \quad (26)$$

Relaxed constraints:

$$mq_3 - \mu_3 - \mu_8 - \mu_5 - \mu_6 - (v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) \leq 0 \quad (27)$$
$$mq_6 - \mu_2 - \mu_7 - \mu_5 - \mu_6 - (v_{13} + v_{14} + v_4 + v_5 - v_9 - v_{10}) \leq 0 \quad (28)$$



n
C9
n
C8
C6
C5
C3
m
p7
t13
m
C7
t14
p8
m
t6
C4
t3
t4
t5
t1
p1
t2
p2
2
m
C2
2
p3
t7
p4
t8
m
C1
t9
t10
t15
t11
t12
p5
p6
t16

$D_c =$

$$\begin{bmatrix}
0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Final Remarks

Computability: This paper has shown that generalized linear constraints can be enforced as effectively as linear marking constraints.

Generality: Generalized linear constraints can describe any supervisor consisting of control places connected to the transitions of a plant PN.

Flexibility: The technique of this paper transforms the problem of enforcing generalized linear constraints into a problem of enforcing linear marking constraints. Any method can then be used to solve the linear marking constraint problem.

Implementation: Software implementation available within the DES software package at: `http://www.nd.edu/~isis/techreports/spnbox/`

References

Giua A. et al, "Generalized Mutual Exclusion Constraints on Nets with Uncontrollable Transitions," 1992 CSMC.

Giua A. and C. Seatzu, "Supervisory Control of Railway Networks with Petri Nets," 2001 CDC.

Li Y. and W. Wonham, "Control of Vector Discrete-Event Systems II - Controller Synthesis," IEEE Trans. Aut. Contr., 39(3), 1994.

Moody J. et al, "Enforcement of event-based supervisory constraints using state-based methods," 1999 CDC.

Moody J. and Antsaklis P., *Supervisory Control of Discrete Event Systems Using Petri Nets*, Kluwer 1998.

Yamalidou E. et al, "Feedback Control of Petri Nets Based on Place Invariants," Automatica 32(1), 1996.