Periodic AC Signals

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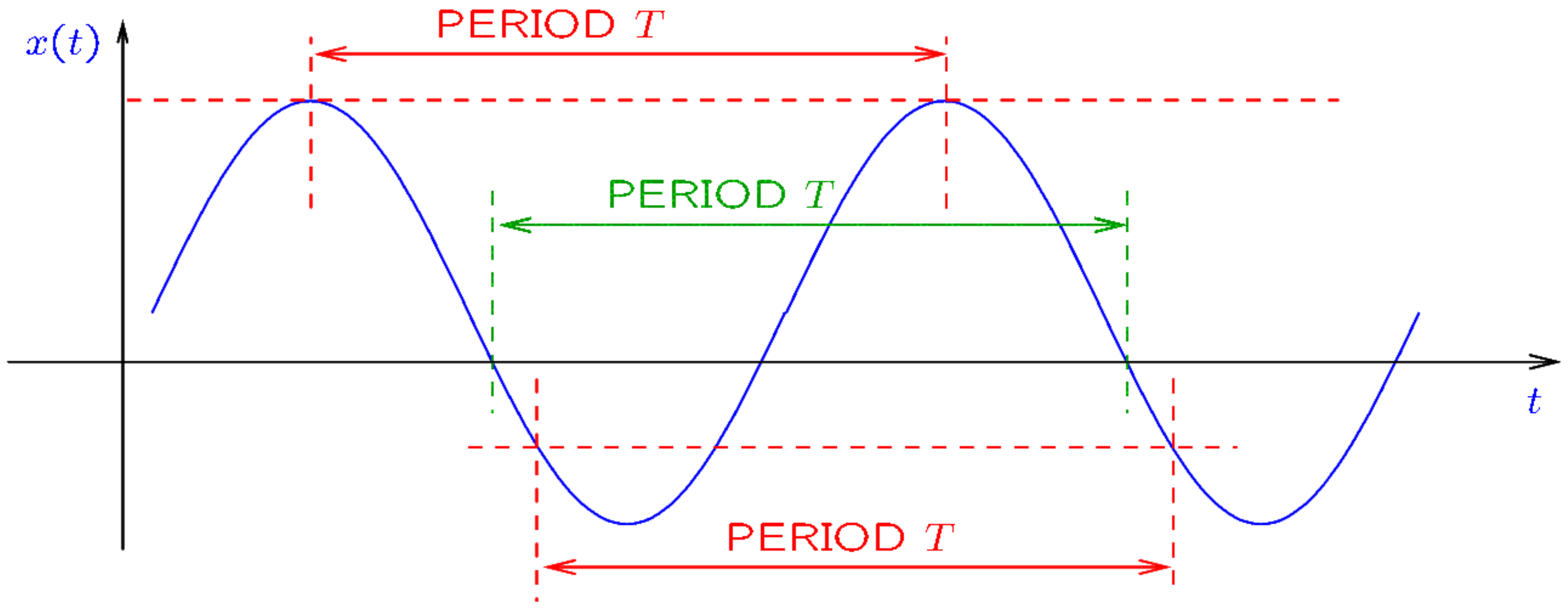
See https://mviordache.name/EEGR2051 for more information.

Frequency and Period

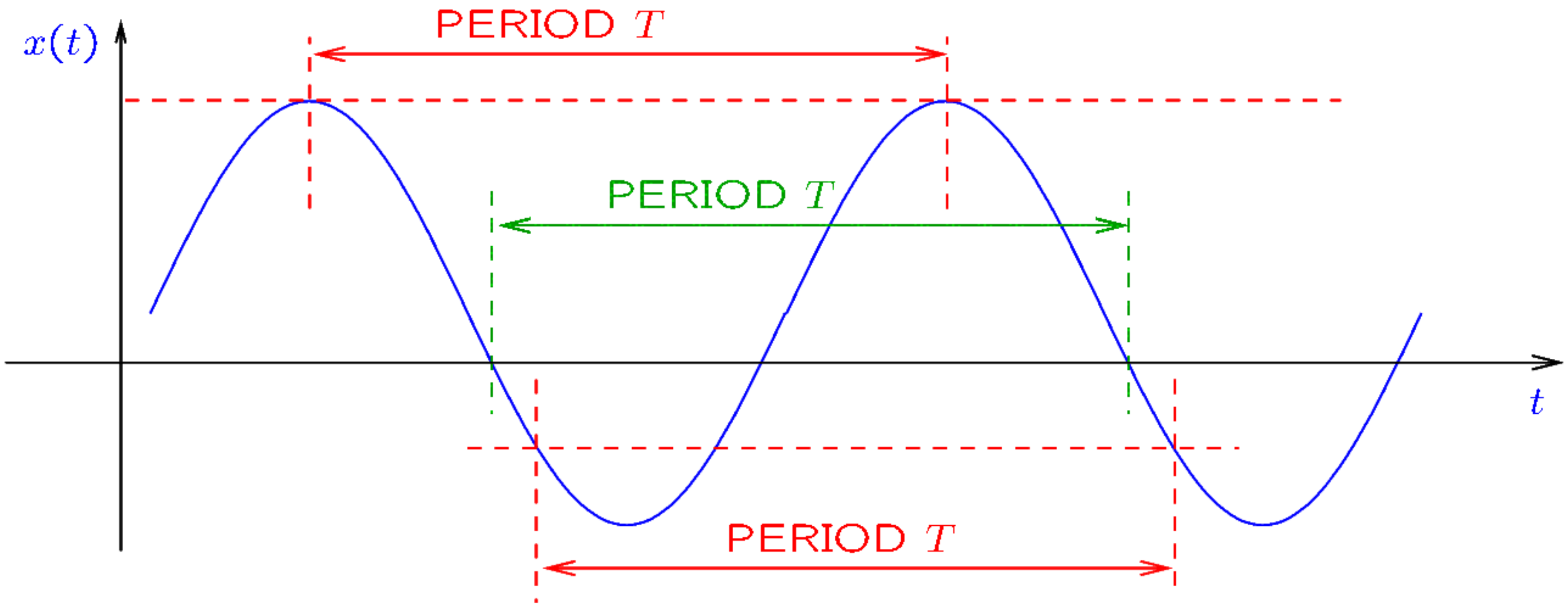
- Voltages and currents can be *periodic* functions of time.
- The period is the duration of one cycle.
- The period can be found from the distance between the intersection points of the curve with a horizontal line:



Frequency and Period

- The frequency is the number of cycles in one second.
  - The unit of frequency is Hertz [Hz].
- The frequency f is related to the period T by the equation

$$f = \frac{1}{T}$$



Frequency and Period

- The frequency f is related to the period T by the equation

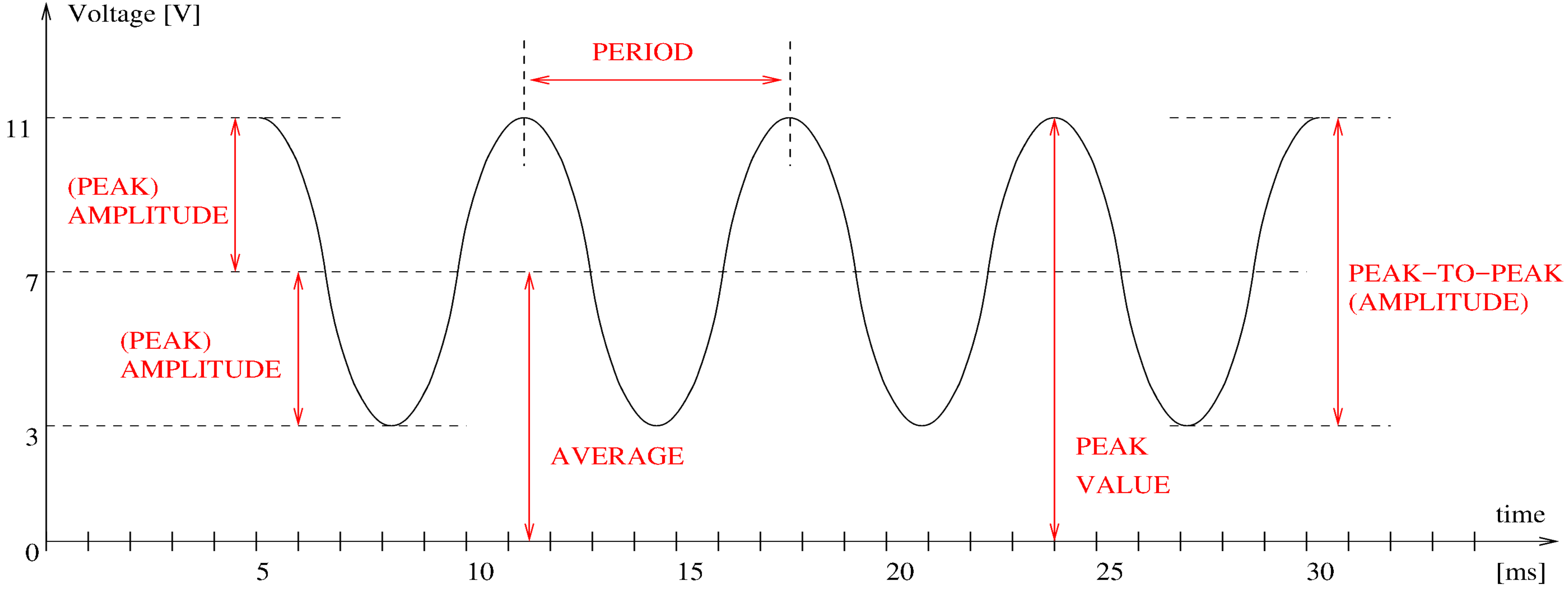
$$f = \frac{1}{T}$$

- The *angular frequency* ω in radians per second is

$$\omega = 2\pi f$$

Peak-to-peak and Peak Amplitudes

- In the example below, the *peak amplitude* is $4\ V$ and the *peak-to-peak amplitude* is $11 - 3 = 8\ V$. The *period* is 6.5 ms and the *frequency* $\simeq 154\ Hz$.

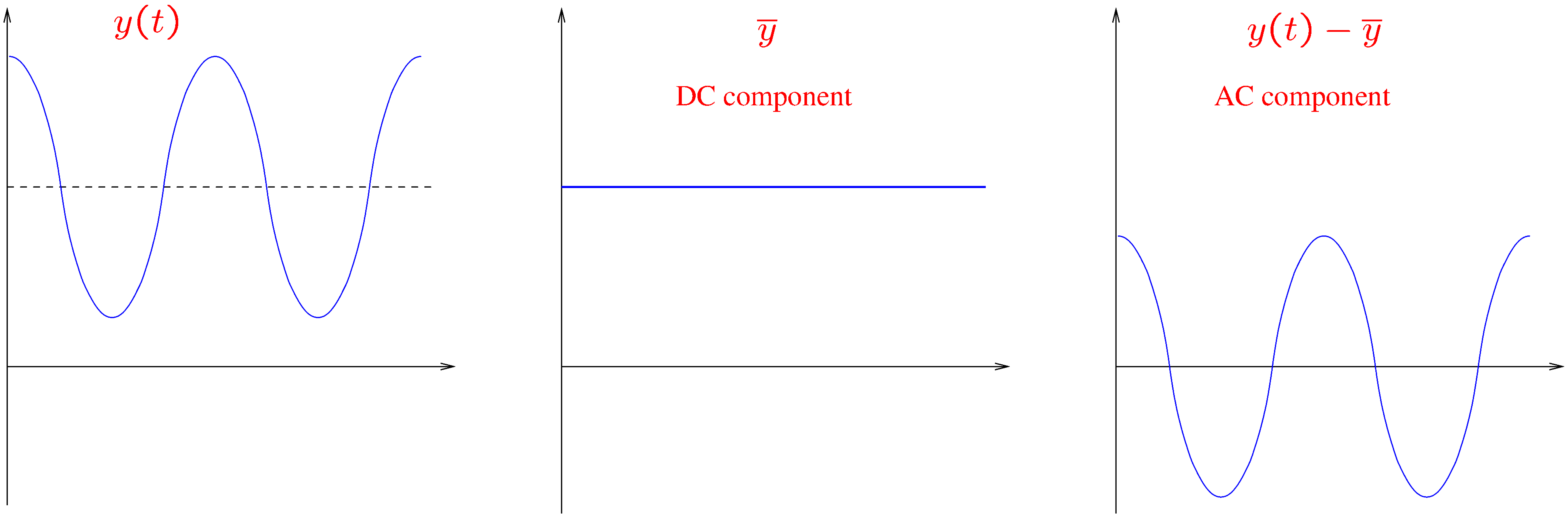


Peak-to-peak and Peak Amplitudes

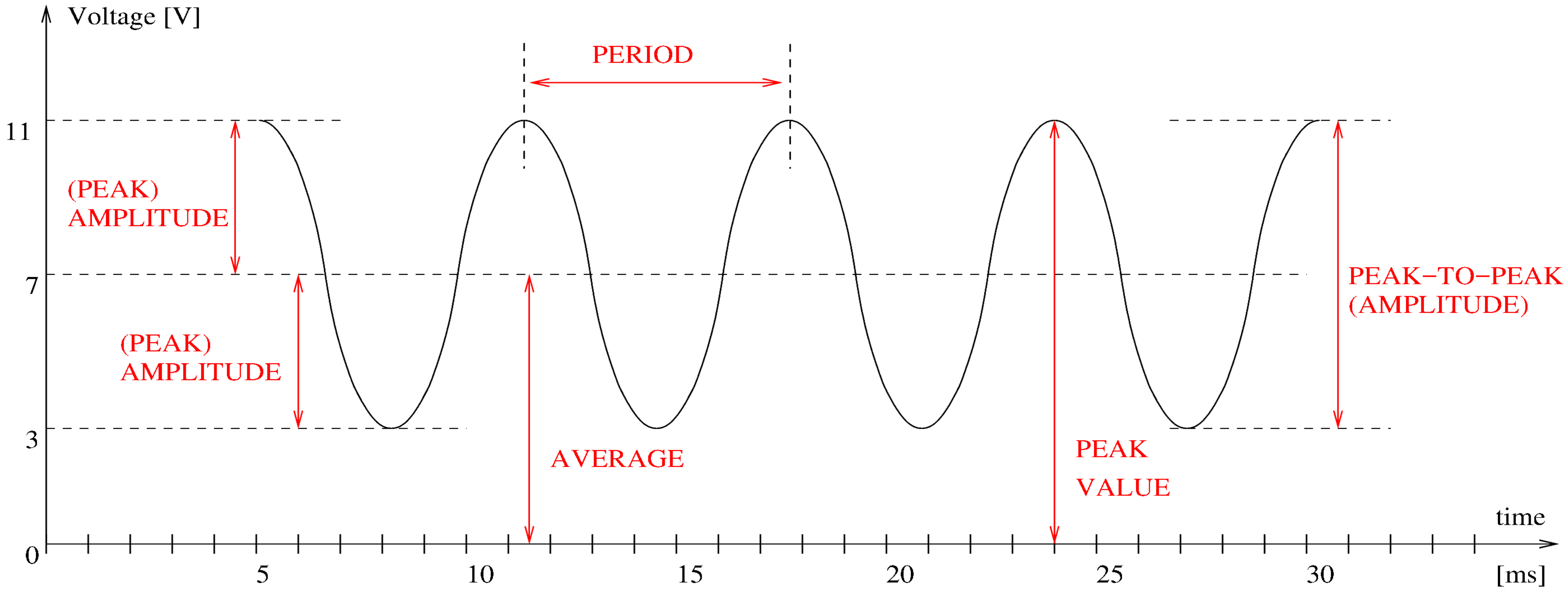
- The *peak-to-peak amplitude* is the difference between the maximum and the minimum values of a signal.
- The *peak amplitude* of a *sinusoidal* signal is the difference between the maximum value and the average value.
  - The peak amplitude also equals the difference of the average value and the minimum value.
- Peak amplitude is defined in more than one way for signals that are not sinusoidal.
- In the context of *sinusoidal* signals, *amplitude* normally means peak amplitude.
- In the context of *arbitrary* periodic signals, *amplitude* commonly means peak-to-peak amplitude.

AC and DC Components

- Let $y(t)$ be an arbitrary signal. Let $\overline{y}$ be the average of $y(t)$.
- Any signal $y(t)$ can be decomposed into a constant (DC) signal of value $\overline{y}$ and a zero-mean alternating (AC) signal of value $y(t) - \overline{y}$.



AC and DC Components—Example



- The *DC component* $\bar{y}$ has $7\ V$ (it equals the average value of the signal).
- The *AC component* $y - \bar{y}$ is a zero-mean sine of $4\ V$ peak amplitude.

AC and DC Components

- To measure the DC component (the average) of a signal with a DMM:
  - Press DCV to measure voltage.
  - Press DCI to measure current.
    - On some DMMs, DCI is SHIFT + DCV.
  - The DMM will display the average value of the signal.

- To measure the AC component with a DMM:
  - Press ACV to measure voltage.
  - Press ACI to measure current.
    - On some DMMs, ACI is SHIFT + ACV.
  - The DMM will display the *RMS value* of the signal.

RMS Voltages and Currents

- The RMS value is a number describing the strength of a signal.
- Note that the RMS value is the DC equivalent of an AC signal.
- This means that *an alternating voltage of RMS value* V_x *and a constant voltage of value* V_x *will produce the same average power on a resistor*.
- Note that average power is proportional to the square of the RMS value.

RMS Voltages and Currents

- Let $u(t)$ be a current or a voltage.
- The *root mean square* value of $u(t)$, also known as *RMS* value or *effective* value is informally defined as

$$U_{rms} = \sqrt{Average\ of\ u^2}$$

- Formally, assuming a signal of period T:

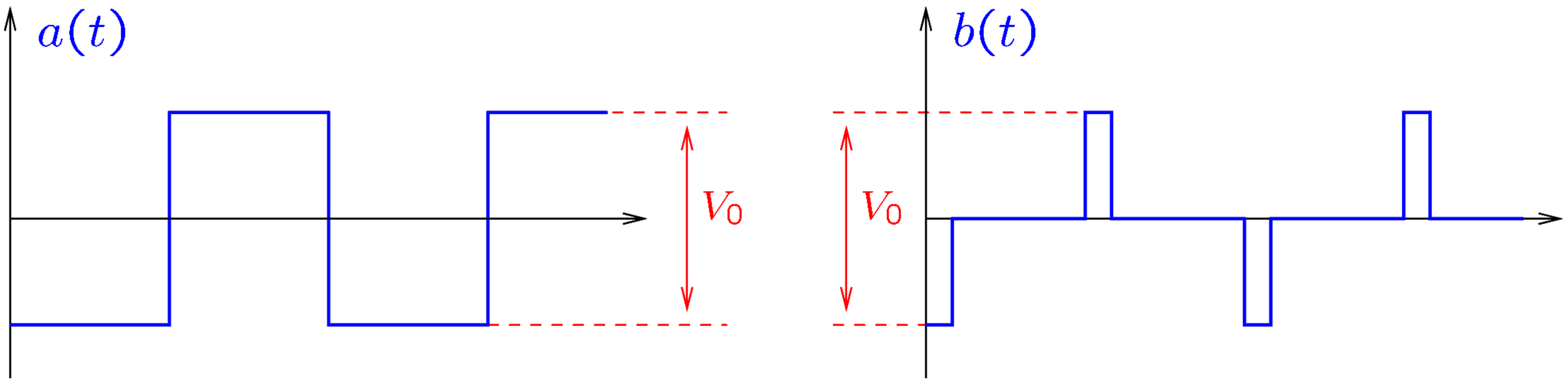
$$U_{rms} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} u(t)^2 dt}$$

- For a *zero-mean sine* of peak amplitude U_m, the formula simplifies to:

$$U_{rms} = U_m/\sqrt{2}$$

Why Do DMMs Display RMS and not Amplitude?

- Amplitude is not a good indicator of the strength of a signal.
- For example, while the signals $a(t)$ and $b(t)$ have the same peak-to-peak amplitude, $b(t)$ is most of the time zero.
- It is clear that $b(t)$ is not as strong as $a(t)$!
- The RMS values do indicate that $b(t)$ is weaker: $B_{rms} \simeq \frac{V_0}{4.8}$, while $A_{rms} = \frac{V_0}{2}$.



RMS Measurements

- Assume a signal $y(t)$ of average value $\overline{y}$.
- In DC mode, the DMM measures the DC component (the average $\overline{y}$).
  - *Press DCV to measure voltage and DCI to measure current.*
    - *On some DMMs, DCI is SHIFT + DCV.*
- In AC mode, the DMM measures the AC component (the RMS of $y - \overline{y}$).
  - *Press ACV to measure voltage and ACI to measure current.*
    - *On some DMMs, ACI is SHIFT + ACV.*
- In AC + DC mode, the DMM measures the RMS of the entire signal $y(t)$.
  - *Press ACV and DCV at the same time to measure voltage.*
  - *Press ACI and DCI at the same time to measure current.*
    - *Some DMMs have a separate button for AC + DC mode.*
- The AC+DC measurement is related to the DC and AC measurements by the formula

$$Y_{rms} = \sqrt{\overline{y}^2 + Y_{ac,rms}^2}$$

Example 1: Find the RMS value of $y(t) = 5\,cos(300t + 40°)\,V$.

- *The peak amplitude is* $Y_m = 5\,V$.
- *Note that the mean of* $y(t)$ *is zero:* $\overline{y} = 0$.
- *For a zero-mean sine the following formula applies:*

$$Y_{rms} = \frac{Y_m}{\sqrt{2}}$$

- *Therefore, the RMS value is* $\frac{5}{\sqrt{2}} \simeq 3.54\,V$.

Example 2: A zero-mean sinewave has $6.4\,V$ *peak-to-peak. Find the RMS value.*

- *The peak amplitude is* $Y_m = \frac{6.4}{2} = 3.2\,V$.
- *Since the sinewave has a zero mean,* $Y_{rms} = \frac{3.2}{\sqrt{2}} = 2.26\,V$.

Example 3: A DMM measures the current $i(t) = 3\,cos(300t - 60°)\ A$. What values should the DMM display in DC, AC, and AC+DC modes?

- *The peak amplitude is $I_m = 3\ A$.*
- *Note that the mean of $i(t)$ is zero: $\bar{i} = 0$ (there is no DC component).*
- *The RMS value is $\frac{3}{\sqrt{2}} \simeq 2.12\,V$.*
- *In DC mode the DMM will display the average, that is, 0 A.*
- *In AC mode the DMM will display the RMS value, that is, $2.12\,V$.*
- *In AC + DC mode the DMM will display the same value as in AC mode, since there is no DC component.*

Example 4: A DMM measures the DC current $I = 3\ A$. What values should the DMM display in DC, AC, and AC+DC modes?

- *In DC mode,* $3\ A$.
- *In AC mode,* $0\ A$*, since there is no AC component.*
- *In AC+DC mode,* $3\ A$*, the same value as in DC mode, since there is no AC component.*

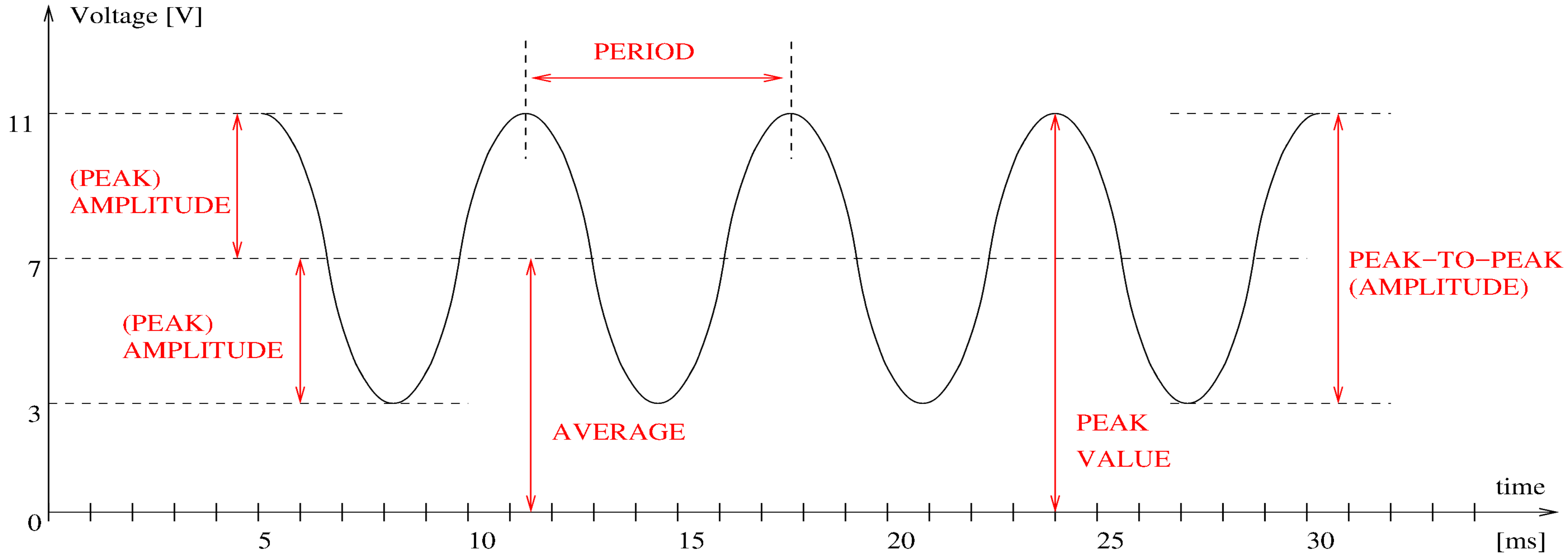
Example 5: A DMM measures $v(t) = 2\cos(100t) + 3\ V$. What values should the DMM display in DC, AC, and AC+DC modes?

- *In DC mode,* $3\ V$*, since the average (the DC component) is* $3\ V$.
- *In AC mode,* $\frac{2}{\sqrt{2}} = 1.41\ V$*, since the AC component is* $2\cos(100t)\ V$.
- *In AC+DC mode,* $\sqrt{3^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 3.32\ V$.

Notation

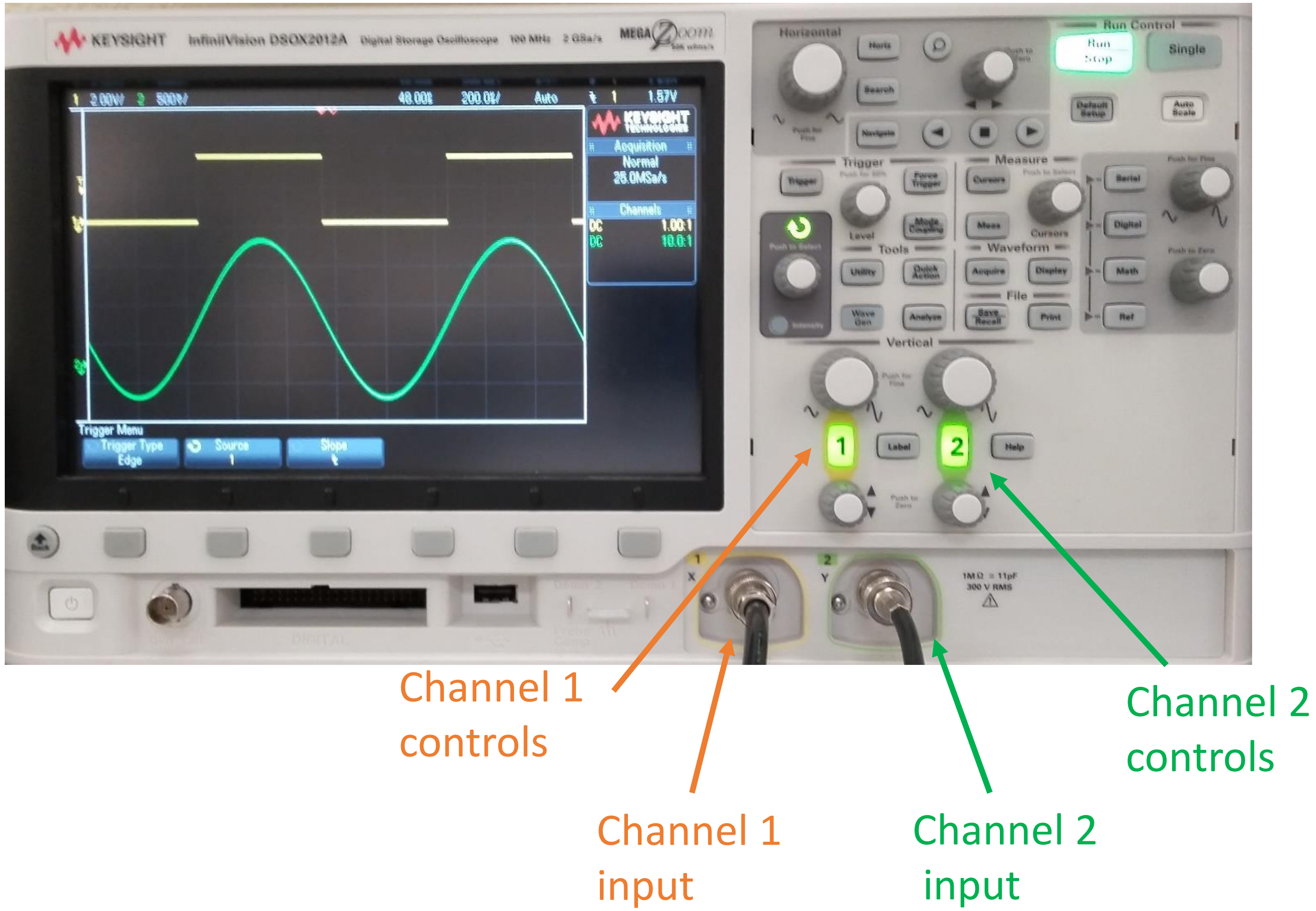
- Volts (or amperes) peak-to-peak are commonly abbreviated V_{pp} (or A_{pp}).
- RMS volts (or amperes) are commonly abbreviated V_{rms} (or A_{rms}).

For example, the waveform in the figure has $8\ V_{pp}$. *Moreover, its AC component has* $\frac{4}{\sqrt{2}} = 2.82\ V_{rms}$.



Visualizing Signals

- The oscilloscope is an instrument that displays signal graphs (voltage versus time).
- Signals on several channels may be displayed at the same time.

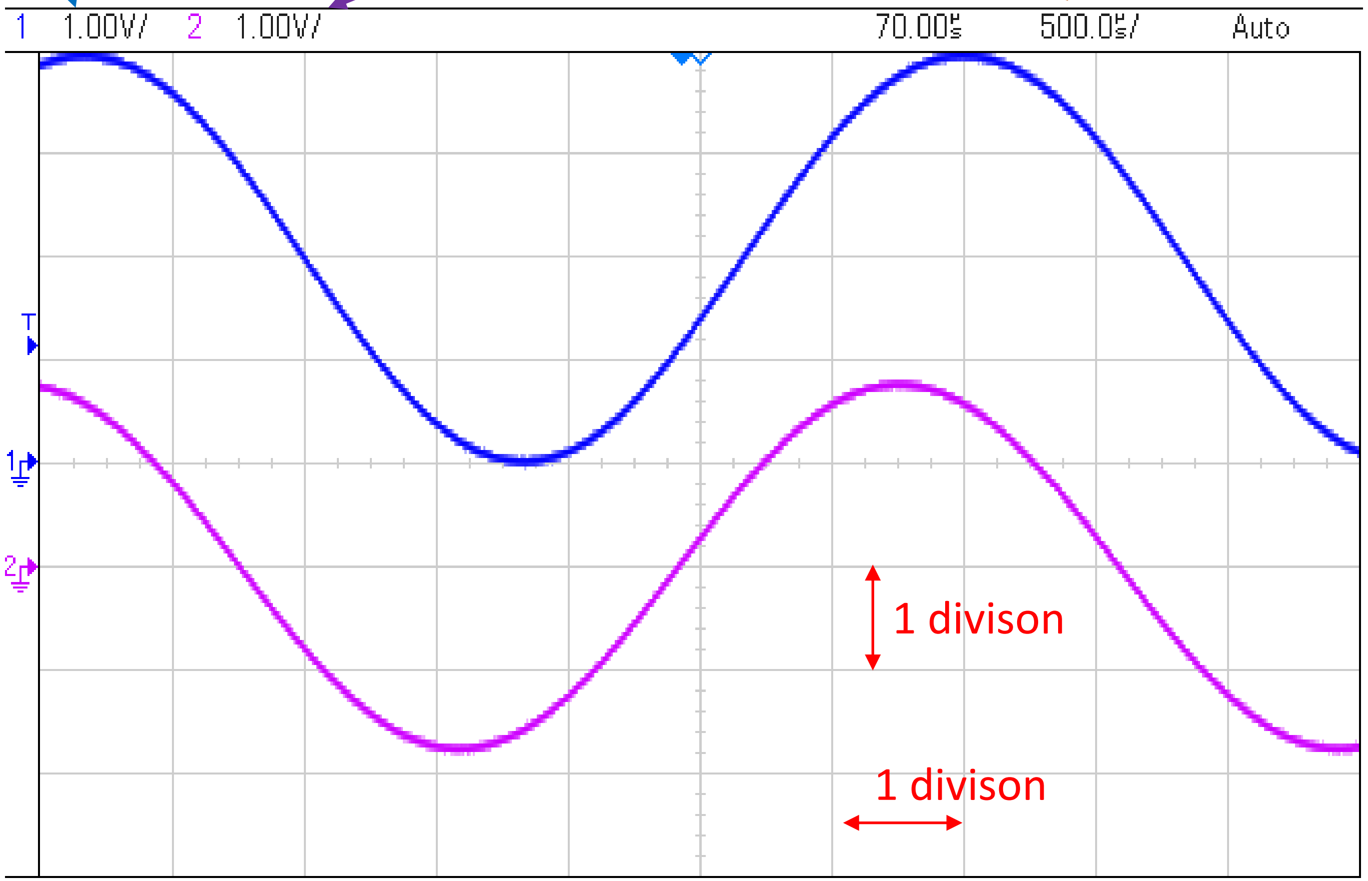


How to Read The Graphs

Channel 1 has 1V/div, that is, 1V per vertical division

Channel 2 has 1V/div

Each horizontal division has 500 μs

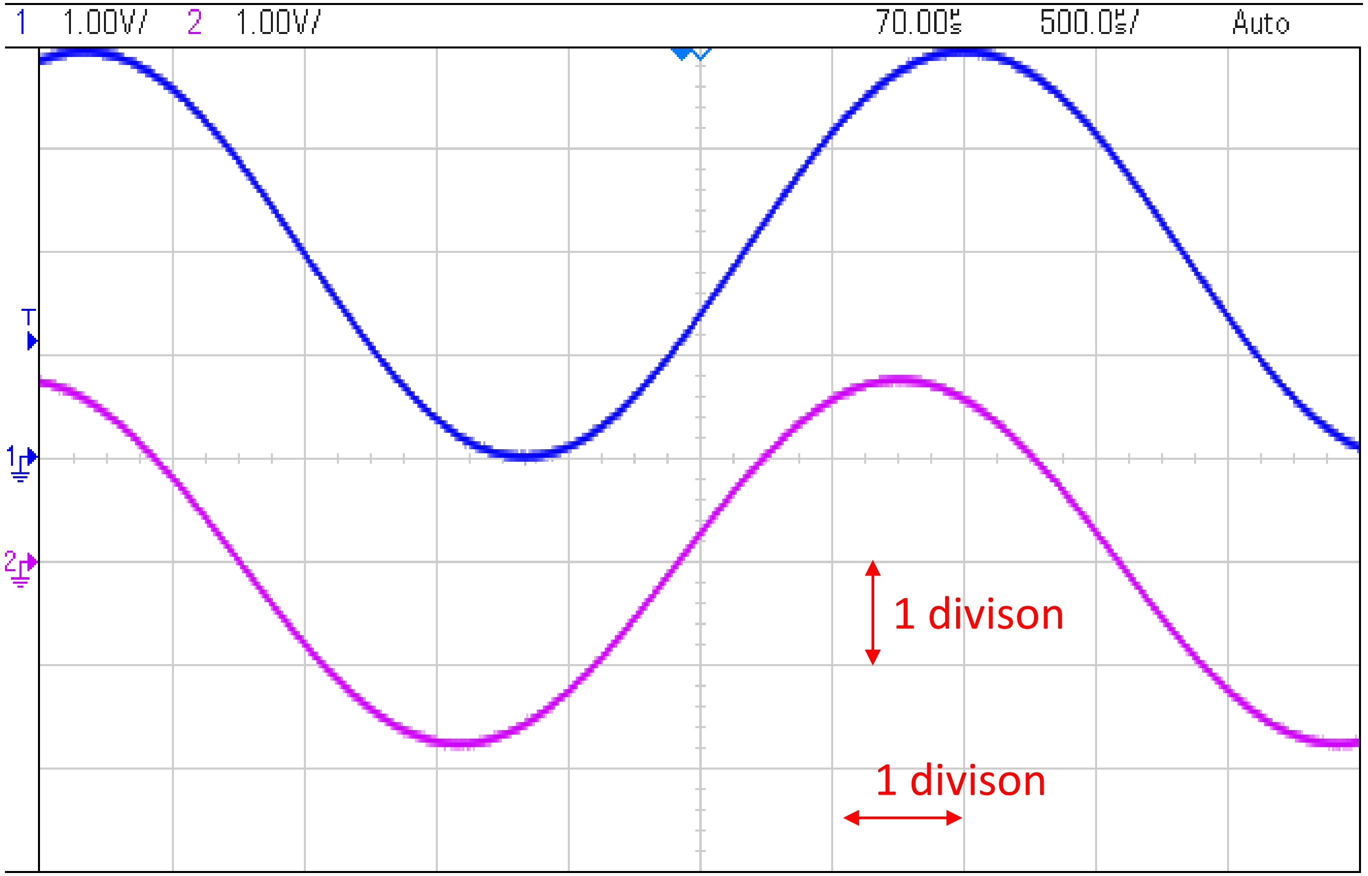


Zero reference of channel 1 (position of ground reference)

Zero reference of channel 2 (position of ground reference)

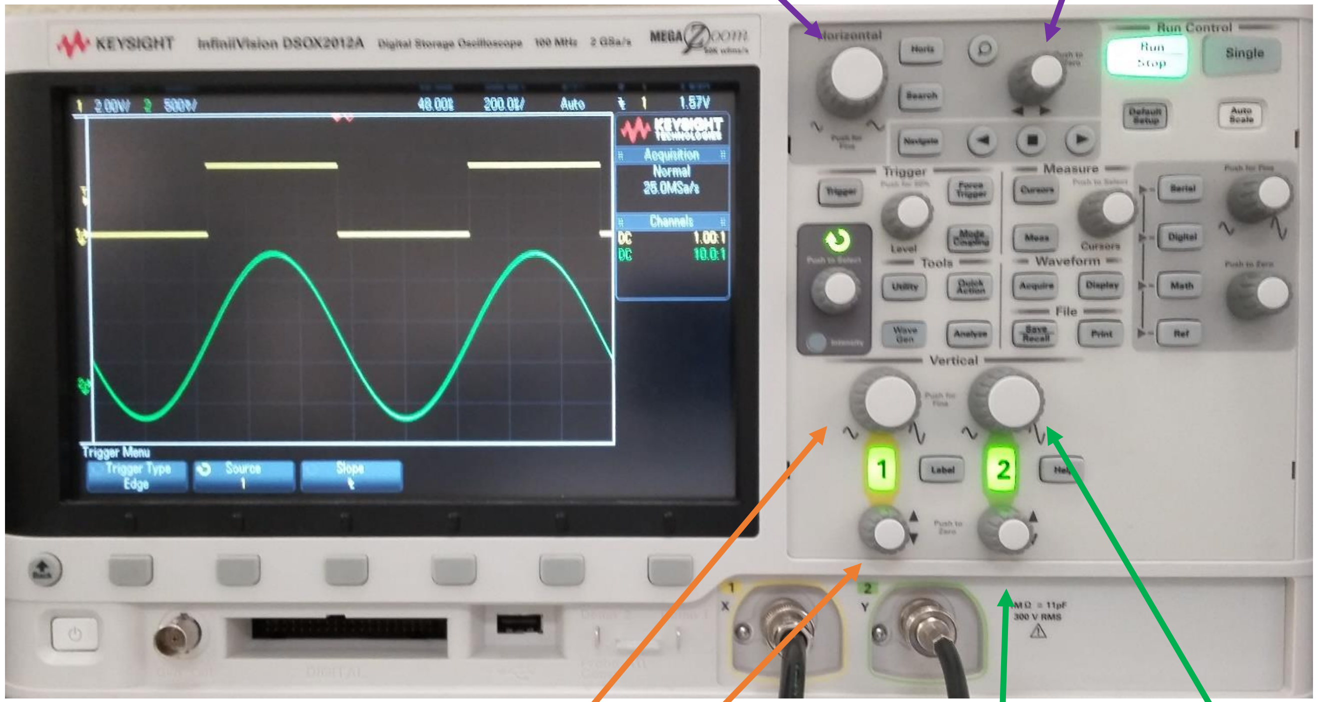
Example

- *The signal on channel 1 (CH1) has a peak-to-peak amplitude of* $4\ div \times 1\frac{V}{div} = 4\ V_{pp}$.
- *The signal on channel 2 (CH2) has a peak amplitude of* $1.8\ div \times 1\frac{V}{div} = 1.8\ V$.
- *The DC component of signal 1 is* $2\ V$.
- *The two signals have a period of* $6.6\ div \times 500\frac{\mu s}{div} = 3.3\ ms$.
- *The frequency is* $\frac{1}{3.3\ ms} \simeq 300\ Hz$.



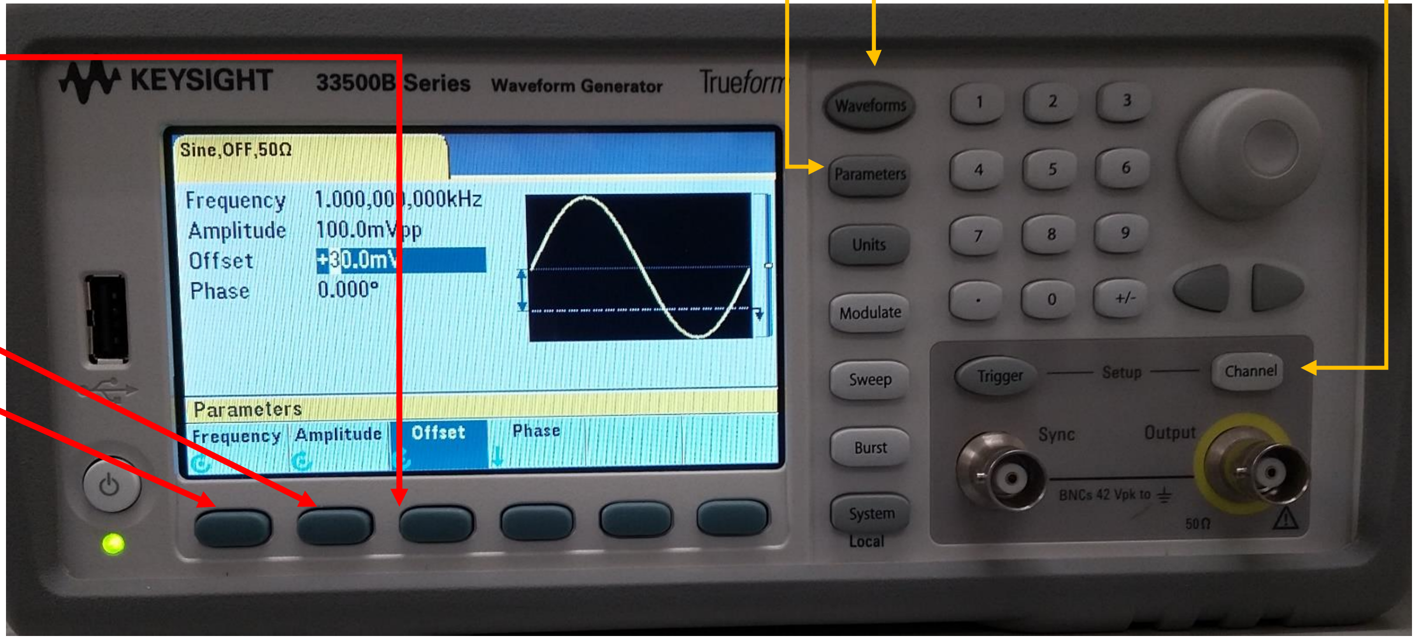
Visualizing Signals

- It is possible to magnify or reduce the size of the graphs by changing the volts per division (that is, *the vertical sensitivity*) and the time per division (that is, *the horizontal sensitivity*).



Generating AC Signals

- Arbitrary waveform generators can be used to generate AC signals.
  - First, select the waveform type (sine, square, …)
  - Next, specify the resistance of the load.
  - Next, specify waveform parameters:
    - Average value (offset)
    - Amplitude
    - Frequency
    - …

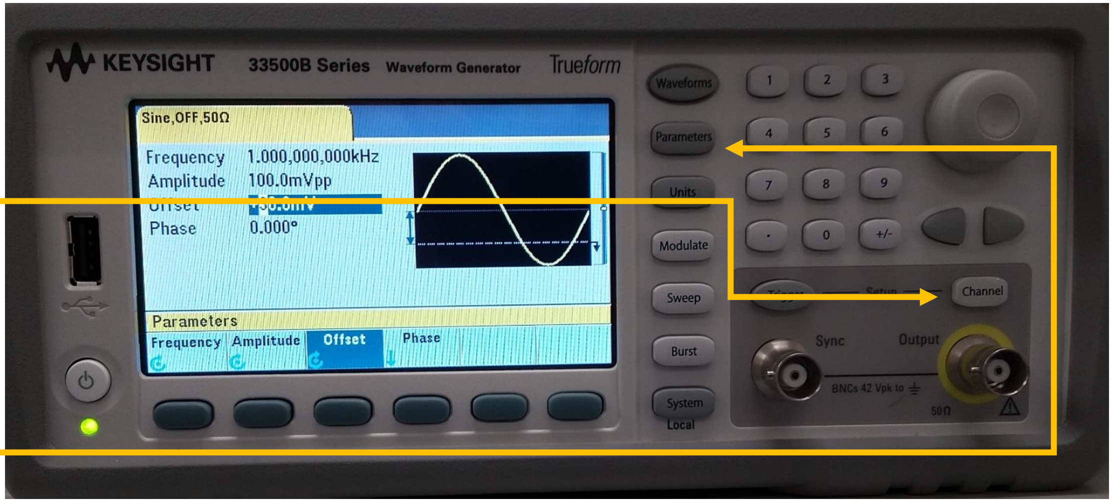


How to Adjust Amplitude

- Waveform generators are unlike ideal sources of voltage in that their internal resistance is not negligible.
  - The internal resistance matches the “characteristic impedance” of common cables used to transmit signals, and its role is to ensure that waveforms can be transmitted without distortion.
- Therefore, the amplitude of the source will depend on the load connected to it!

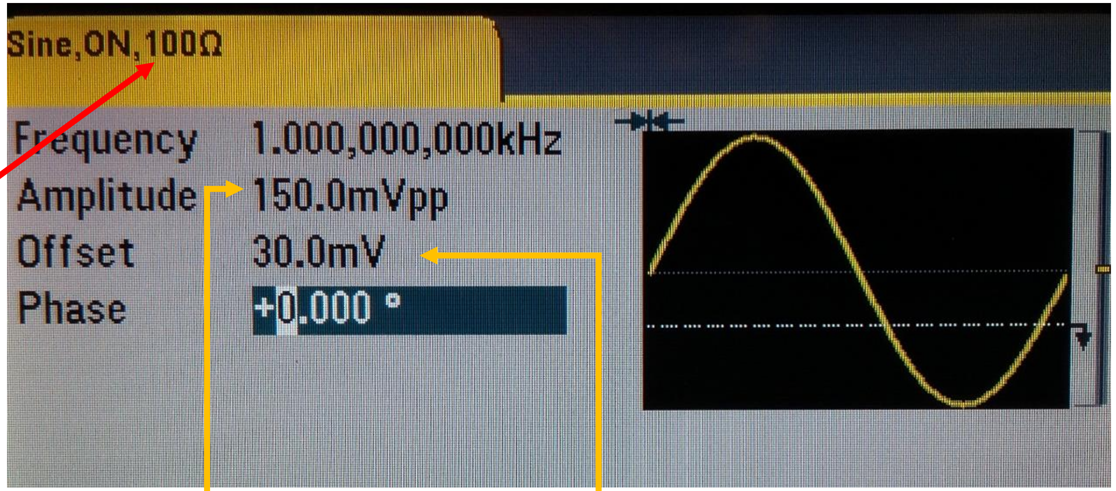
For example, a source with an internal resistance of $50\ \Omega$ *that outputs* $4\ V$ *on a* $100\ \Omega$ *load, will output* $4.8\ V$ *on a* $200\ \Omega$ *load and* $2\ V$ *on a* $25\ \Omega$ *load.*

- To adjust the amplitude or the offset of the source:
  - First, specify the resistance of the load (press *Channel*, then *Output Load*.)
  - Next, specify the amplitude and offset that the source should have when driving a load that has the specified resistance (press *Parameters*.)



Example

Assume the settings shown in the picture and an internal resistance $r = 50\ \Omega$. If the source powers a resistor $R = 200\ \Omega$, what is the output voltage and the offset?



- The frequency is $f = 1\ kHz$.
- The angular frequency is $\omega = 2\pi f = 6283.2\ rad/s$.
- The specified load resistance is $R_L = 100\ \Omega$.
- The output amplitude can be found by a method known as *voltage division*.

$$V_s = 150 \cdot \frac{R_L + r}{R_L} \cdot \frac{R}{R + r} = 180\ mV_{pp}$$

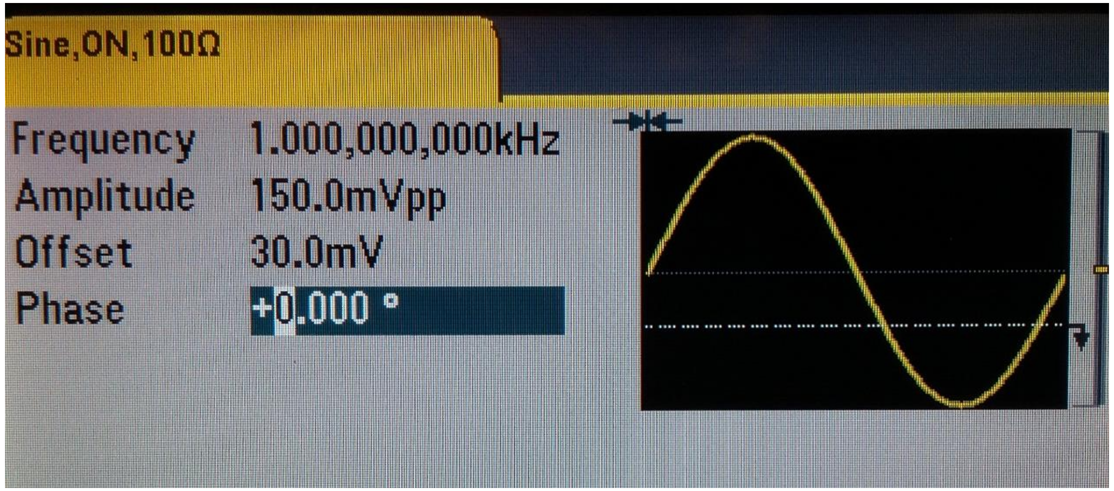
- The output offset is

$$V_o = 30 \cdot \frac{R_L + r}{R_L} \cdot \frac{R}{R + r} = 36\ mV$$

- The output signal is $v(t) = \frac{180}{2} \cdot \sin(6283.2t) + 36 = 90 \cdot \sin(6283.2t) + 36\ mV$.

How to Adjust Amplitude

- *If the load is not equivalent to a resistor or if the resistance of the load does not equal the specified value of the output load, the amplitude and the offset displayed by the source should be ignored, as they will be different from the actual amplitude and offset of the signal.*



Adjusting Amplitude—The Easy Way

- To adjust the amplitude or the offset of the source:
  - Measure its value with an oscilloscope or a DMM.
  - Adjust it with the rotary knob and the arrow keys until its measured value is right.